How Central Banks Should Use Their Balance Sheets to Control Flight-to-Safety

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Abstract

I study how central banks should use their balance sheets to control flight-to-safety in a model with retail and wholesale banks, where the magnitude of flight-to-safety is endogenously determined by the severity of wholesale banking panics. I show that expanding the central bank's balance sheet, by purchasing government bonds through issuance of reserves, mitigates wholesale banking panics. However, this policy lowers asset returns, limiting the usefulness of assets in transactions, which harms depositors. Instead, swapping reserves for other central bank liabilities accessible to wholesale banks, for instance, through the U.S. Federal Reserve's overnight reverse repurchase agreement facility, mitigates panics without reducing asset returns or transactions. Critical for these results are endogenous shifts in asset demand. When central banks reduce the supply of safe government bonds to private markets, as in balance sheet expansions, there are large increases in the demand for private banks' liabilities which bid down their returns. Adjusting the composition of central bank liabilities avoids such large demand shifts.

Key Words: central bank balance sheet, flight-to-safety, wholesale bank
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1 Introduction

In periods of financial distress, investors reallocate their portfolios from riskier assets toward safer ones. These *flight-to-safety* phenomena reflect large shifts in asset demand during financial crises. In particular, panics in the wholesale banking sector, which is beyond the coverage of financial regulations and supervision, have driven several flight-to-safety events and are viewed as a major source of financial instability in past decades (Bernanke, 2012, 2018; Gorton, 2010).[1] For example, during the 2008 Financial Crisis, the failure of Lehman Brothers sparked a panic in money market mutual funds, with over $400 billion withdrawn in September 2008. More recently, during the COVID-19 pandemic, investors moved over $100 billion from prime money funds to safe assets backed by government securities in March 2020 (Sengupta & Xue, 2020).

I study the implications of central bank balance sheet policies during flight-to-safety episodes triggered by wholesale banking panics.[2] The key finding is that *central banks can mitigate flight-to-safety by expanding their balance sheets, but this lowers asset returns, reducing the usefulness of assets in transactions and potentially harming welfare.* In essence, an open market operation that expands the balance sheet by purchasing Treasury bills stabilizes short-term funding markets because it lowers the returns on these safe securities, reducing investors' incentive to seek refuge in them. However, the decreased returns hinder transactions involving Treasury bills, like repo transactions. I show that the return reduction holds broadly across assets, even for risky bank liabilities. The key is the *endogenous shifts in asset demand*: by mitigating flight-to-safety, investors switch to bank liabilities, bidding down their returns and making them less useful in transactions.

[1] Wholesale (or shadow) banks, such as money market funds, are financial institutions engaging in "credit intermediation involving entities and activities outside the regular banking system" (Financial Stability Board). In 2022, these banks held $63 trillion, representing 79% of global GDP (S&P Global).

[2] Existing regulatory tools for financial stability are limited in their effectiveness and scope of coverage in wholesale banking. Moreover, wholesale banks often fall beyond the reach of direct central bank crisis interventions. For example, the Dodd-Frank Act removed the U.S. Federal Reserve's authority to lend to wholesale banks (Fischer, 2016). Recently, attention has been drawn to the financial stability implications of central bank balance sheet policies (Bernanke, 2016; Greenwood, Hanson, & Stein, 2016).

Specifically, I develop a three-period, two-sector banking model with retail and wholesale banks that conduct intermediation activities to support depositors' transactions. Unlike retail banks, wholesale banks are not subject to stringent regulations, such as the leverage requirements in Basel III, and have no reserve account at the central bank.[3] They serve institutional investors, who often conduct large-scale transactions with government bonds, rather than individual businesses and consumers who trade with bank deposits. Linking the two sectors is an interbank market that allows wholesale banks to lend to retail banks, providing a conduit for indirect access to central bank reserves.

Crucially, panics arise in the wholesale banking sector due to the risk of banking failure, as in Gertler and Kiyotaki (2015). Depositors anticipate potential losses, withdraw deposits, and flee to safe government bonds that are extensively used in wholesale payments. Their endogenous withdrawal behavior captures the severity of wholesale banking panics or the magnitude of flight-to-safety. I evaluate the general equilibrium effects of central bank balance sheet expansions, which not only alter the supply of bank deposits and government bonds but also depositors' withdrawal behavior, shifting their demand between these assets.

I show that an expansion in the size of the central bank's balance sheet, through open market operations, mitigates wholesale banking panics. A balance sheet expansion involves purchasing government bonds with the issuance of new reserves. This reduces the supply of bonds in the private sector, lowering their interest rate (increasing their price) and making bonds a less attractive safe harbor. Simultaneously, the balance sheet expansion increases the reserve supply, facilitating transactions in the interbank market, where retail banks borrow from wholesale banks using reserves as collateral. In this way, this expansion makes it easier for wholesale banks to invest in interbank loans, allowing them to issue more attractive deposits. Overall, the balance sheet expansion mitigates banking panics by making government bonds less attractive relative to wholesale deposits.

[3] Similar definitions are used by Gertler, Kiyotaki, and Prestipino (2016) and Ordoñez (2018).

Despite mitigating banking panics, the central bank balance sheet expansion lowers the returns obtained by retail and wholesale bank depositors, limiting transactions. First, this expansion reduces the supply of government bonds in the private sector, lowering their return and harming wholesale bank depositors who use them in wholesale payments. Second, by mitigating banking panics, a large number of wholesale bank depositors shift demand away from government bonds toward bank deposits, bidding down deposit returns, even though the balance sheet expansion promotes deposit issuance. Finally, this expansion also reduces the return on retail bank deposits. Retail banks pay lower returns on deposits because, otherwise, they could substitute their funding source from deposits to cheaper interbank loans. Wholesale banks ask for lower returns on interbank loans due to their higher asset demand to back their increased deposit liabilities.

To better understand the crucial role of depositors' asset demand, I evaluate balance sheet expansions in extreme cases where depositors endogenously choose not to shift their asset demand. This occurs when wholesale bank depositors invariably expect a higher return on either deposits or government bonds, strictly preferring one over the other. Unlike before, balance sheet expansions do not necessarily harm depositors. The increased reserve supply promotes the issuance of bank deposits, at least by retail banks. With no shift in depositors' asset demand, the increased deposit supply implies higher (instead of lower) deposit returns.

Finally, I study a balance sheet policy that alters the composition of central bank liabilities without altering the size of the balance sheet, and therefore, does not change the supply of government bonds. To do this, I introduce another central bank liability that plays a role similar to the U.S. Federal Reserve's overnight reverse repurchase agreement facility (ON-RRP henceforth), which is accessible to a broader range of financial institutions, including wholesale banks.[4]

[4]Unlike government bonds, ON-RRPs only circulate among financial institutions, so introducing them will not amplify financial instability by providing another safe harbor to depositors.

I show that, like central bank balance sheet expansions, a swap of reserves for ON-RRPs mitigates wholesale banking panics. As more ON-RRPs become available, wholesale banks reduce their reliance on indirect access to reserves through interbank lending. This eases their burden of compensating for retail banks' balance sheet costs arising from stringent regulations.[5] As a result, wholesale banks can offer more attractive deposits to compete for depositors, mitigating banking panics.

Unlike central bank balance sheet expansions, swapping reserves for ON-RRPs increases asset returns and benefits retail and wholesale bank depositors. By mitigating banking panics, this swap reduces wholesale bank depositors' demand for safe government bonds, raising their rate of return. In response to the increased bond return, wholesale banks offer higher returns on deposits to maintain their attractiveness. As wholesale banks incur higher payments on deposit liabilities, they seek higher returns on interbank loans. Consequently, retail banks substitute their funding sources from loans to deposits, and the increase in the supply of retail bank deposits raises their rate of return in equilibrium.

Related Literature The two-sector banking model, to which I introduce financial instability in the form of wholesale banking panics, builds on Williamson (2019). These panics arise from random bank failures and depositors' lack of information about which banks will fail — distinct from panics induced by sequential service constraints (i.e., first-come-first-served), as in Diamond and Dybvig (1983).[6] Gertler, Kiyotaki, and Prestipino (2016) study wholesale banking panics in a general equilibrium framework and explore the role of lender-of-last-resort and macroprudential policies. I instead focus on central

[5]Specifically, the balance sheet costs arise from retail banks' leverage requirements, which restrict their liability-to-asset ratios and force them to conduct costly equity financing. Kim, Martin, and Nosal (2020), Martin, McAndrews, Palida, and Skeie (2019), and Williamson (2019) also show how the costs associated with stringent banking regulations limit banks' ability to grow their balance sheet and provide useful financial intermediation activities.

[6]As shown by Andolfatto and Nosal (2020) and Huang and Keister (2024), self-fulfilling wholesale banking panics can occur without such constraints. The way I model banking panics arising from uncertainty about the identities of weak financial institutions is consistent with Gorton (2008).

bank balance sheet policies and contribute to a growing literature on the financial stability implications of these policies, including Bernanke (2016), Greenwood, Hanson, and Stein (2016), and Woodford (2016), among others.

I show how central bank interventions enhance the stability and resilience of the financial system during banking panics. Andolfatto, Berentsen, and Martin (2020), Cooper and Corbae (2002), and Robatto (2019) show that the central bank can eliminate banking panic equilibria in models with multiple equilibria. This differs from my model, where equilibrium is unique, and the central bank instead controls the magnitude of banking panics through its balance sheet policies. I formalize ideas, such as those in Bush, Kirk, Martin, Weed, and Zobel (2019), that argue an ample reserve supply helps banks meet their asset needs, avoiding damaging effects during crises. Carlson, Duygan-Bump, Natalucci, Nelson, Ochoa, Stein, and Van den Heuvel (2016) show that central bank balance sheet policies can mitigate key threats to financial stability and highlight the usefulness of reserves and ON-RRPs, which aligns with my findings.

A challenge in evaluating central bank crisis interventions is that their effects on flight-to-safety, or more generally on financial stability, are entangled with their effects on the supply of safe assets. Safe government bonds, to which depositors might flee, play a critical role in my results. To disentangle these two effects, I study both central bank balance sheet expansions and swaps of reserves for ON-RRPs. While both interventions alleviate flight-to-safety, only balance sheet expansions change the supply of government bonds. Crucially, I show that it is the decreased bond supply that drives the large shifts in asset demand in response to balance sheet expansions, ultimately leading to a general reduction in asset returns on safe government bonds and risky bank liabilities. By contrast, the swaps of central bank liabilities do not withdraw government bonds from the private sector, thereby generating only moderate shifts in asset demand and raising asset returns.

The comparison of the two policies discussed above highlights the key insight of this paper: endogenous shifts in asset demand can generate different, and sometimes, unintended policy implications. This insight complements papers that evaluate central bank interventions under banking panics driven by fundamental risks, such as Allen, Carletti, and Gale (2014), Allen and Gale (1998), Diamond and Rajan (2006), and Robatto (2017, 2024). Again, endogenous shifts in asset demand drive the general reduction in asset return in response to central bank balance sheet expansions. The reversal of asset return and welfare implications between balance sheet expansions and swaps of reserves for ON-RRPs further demonstrates that the magnitude of these demand shifts is not only meaningful but also critical for evaluating central bank crisis interventions.

I focus on short-term funding markets and examine the immediate consequences of policy interventions, as in Gorton and Metrick (2012) and Martin, Skeie, and Thadden (2014). The mechanisms I highlight regarding the mitigation of banking panics and consequences on asset returns and transactions complement those that emphasize either the costs of inefficient liquidation of productive assets (Diamond & Dybvig, 1983) or the long-run benefits of reducing credit costs for production through balance sheet policies, such as quantitative easing (Cardamone, Sims, & Wu, 2023; Cui & Sterk, 2021).

The rest of the paper is organized as follows. I present the model in section 2. In section 3, I solve three types of equilibrium with reserves as the only form of central bank liability and show how the central bank's balance sheet size determines the equilibrium type. Section 4 is an analysis of ON-RRPs. Section 5 concludes.

2 Model

This is a two-sector banking model with retail and wholesale banks. There are three periods, $t = 1, 2, 3$, with no time discounting between periods. There are three sets of private agents: a measure one of depositors, a measure one of producers, and an infinite measure of bankers who self-select into operating retail or wholesale banks. Private agents can work (h) and consume (c). A linear production technology allows them to convert labor to goods one-for-one. There is also a government consisting of a fiscal authority and a central bank.

Depositors work and deposit in period 1, consume in period 2, and work to pay taxes in period 3. Their preferences are captured by $-h_1+u(c_2)-h_3$, where u is strictly increasing, strictly concave, and twice continuously differentiable, with $u(0)=0$, $\lim_{c\to 0} u'(c)=\infty$, $\lim_{c\to\infty} u'(c)=0$, $\lim_{c\to 0} cu'(c)=0$, and $-c\frac{u''(c)}{u'(c)}<1$ for $c\geq 0$. Producers and bankers are risk-neutral and profit-maximizing. Producers work to produce consumption goods for depositors in period 2 and consume returns in period 3. Their payoffs are $-h_2+c_3$. Bankers work in period 1 to raise equity (i.e., sweat equity) and in period 3 to pay off their debts. They consume their profits in either period, and their payoffs are $c_1^B-h_1^B+c_3^B-h_3^B$.

Trade between depositors and producers takes place in bilateral exchanges in period 2, where depositors make a take-it-or-leave-it offer to the producer they meet in exchange for consumption goods, similar to the decentralized market in Lagos and Wright (2005). The reason depositors do not consume the goods they produced in period 1 is that all goods are perishable and cannot be carried across periods. Depositors must also acquire assets in advance, such as deposit claims, to settle their transactions because private agents are subject to limited commitment, implying that unsecured IOUs are not accepted. They acquire assets in period 1, where a centralized Walrasian market allows agents to trade goods and assets. Banks also write deposit contracts with depositors in this period. In period 3, all debts are redeemed, and agents consume their returns.

There are two underlying assets: central bank reserves and government bonds. Central bank reserves are private banks' account balances with the central bank, while government bonds are issued by the fiscal authority. Both assets are issued in period 1. In period 3, the central bank pays a *real interest rate* r^m-1 on reserves, and the fiscal authority pays a *real interest rate* of r^b-1 on bonds.[7] Crucially, reserves and government bonds are not perfect

[7]I focus on the endogenous real interest rates exclusively throughout the analysis, as this paper mainly studies the real effects of central bank balance sheet policies that alter the supply of central bank liabilities and government bonds. In practice, the *nominal* interest rate on reserves is administered by the central

substitutes: reserves are limited to retail banks, whereas anyone, including depositors and producers, can hold government bonds. In addition to the underlying assets, banks have access to a collateral technology that allows them to borrow against assets, creating useful financial intermediary liabilities, such as deposit claims and interbank loans.

2.1 Banking

The two-sector banking structure is similar to Williamson (2019). Among the entire population of depositors, a fraction $\alpha \in (0, 1)$ are retail bank depositors who represent individual consumers and businesses, and the remaining $1 - \alpha$ are wholesale bank depositors who represent institutional investors. These fractions, α and $1 - \alpha$, determine the size of each sector. Retail bank depositors always use deposit claims in trading. However, wholesale bank depositors may have to withdraw from their banks at the end of $t = 1$ when they face trading opportunities that require government bonds, i.e., the producer they meet only accepts bonds.[8] This happens for an exogenous fraction ρ of wholesale bank depositors, and the remaining fraction can use either wholesale banks' deposit claims or bonds in trading.

I introduce instability in the wholesale banking sector, triggered by the risk of banking failure, similar to Gertler and Kiyotaki (2015) and Williamson (2022). This gives rise to another rationale for withdrawals. Specifically, an exogenous fraction $1 - \delta$ of wholesale banks fail at $t = 2$, modelled as a collapse in their collateral technology. Failed banks default on their liabilities and abscond with assets, without incurring any consequences. Despite becoming public information at $t = 2$, the identity of these banks is ex-ante

bank, while the real interest rate is determined by the fundamentals via the inflation rate. I could introduce money (cash) into the model to allow the central bank to set the nominal interest rate on reserves, but the main results of this paper would remain unchanged.

[8]In practice, wholesale payments often involve bonds. In particular, overnight repurchase agreements (repos), for which government bonds are the dominant collateral, involve a large volume of transactions. Repo markets indirectly support exchanges in goods and services at the wholesale level. Directly exchanging bonds is a convenient shortcut to capturing wholesale payments supported by repo markets.

unknown to depositors at $t = 1$. Therefore, the fraction $1-\rho$ of wholesale bank depositors who still have their deposit claims at the end of $t = 1$ can become panicky because they must make their withdrawal decision with imperfect information before any bank fails. These panicking depositors withdraw with an endogenous probability η and compete for government bonds with the remaining ρ of wholesale bank depositors who must trade with bonds.[9]

In equilibrium, the endogenous withdrawal probability η also represents the fraction of panicking depositors who withdraw and flee to safe government bonds, capturing the severity of wholesale banking panics or, equivalently, the magnitude of flight-to-safety. The more severe the panics are, the higher the aggregate demand for bonds, as more depositors flee to these safe government securities. Figure 1 summarizes the timing of events and provides a visual guide to the pattern of meetings and exchanges among private agents.

In addition to the risk mentioned above, retail and wholesale banks differ in the regulatory requirements to which they are subject and in their access to interest-bearing central bank liabilities. Retail banks resemble highly regulated depository institutions. They have access to central bank reserves and are subject to a leverage requirement that limits their liability-to-asset ratio to $\theta \in (0, 1)$. Wholesale banks resemble less-regulated financial institutions like money market funds and hedge funds. They do not have access to central bank reserves and are not subject to leverage requirements. An interbank market links these two banking sectors, which allows wholesale banks to hold reserves indirectly by lending to retail banks.

[9] The collapse in collateral technology provides a convenient shortcut for modelling wholesale banking failure. In a more fully specified dynamic model, I could let a fundamental shock, such as a sunspot shock or a productivity shock, trigger banking failure probabilistically and, eventually, banking panics. However, this would not change my analysis, which relies on the fact that the risk of banking failure generates banking panics, regardless of the underlying shock. Moreover, as in Williamson (2022), the fundamental shock may arise even after deposits are made.

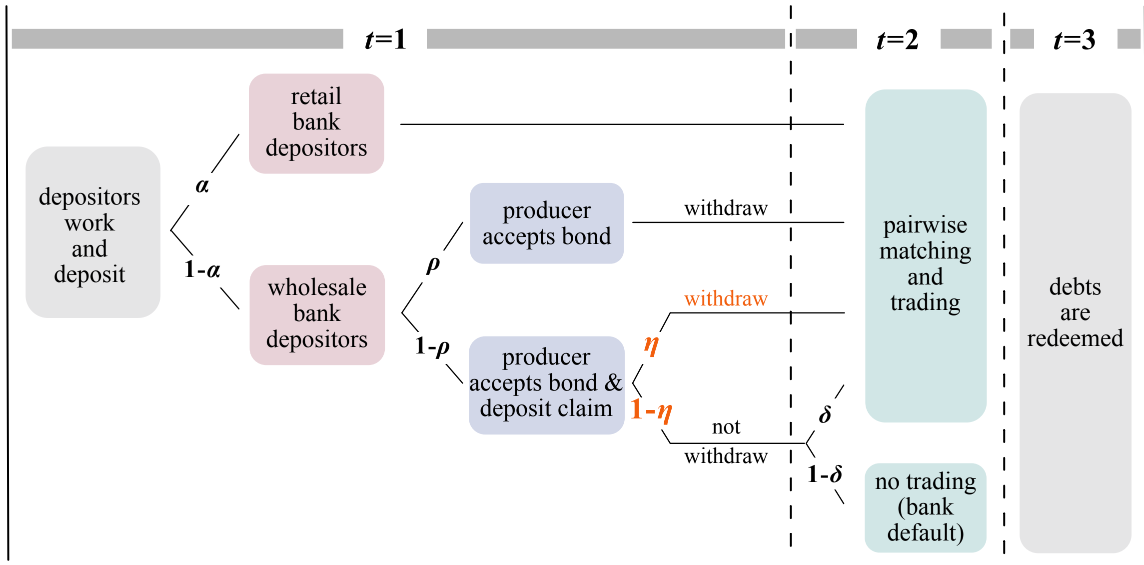

Figure 1: Timing of Events

Retail Banks Retail banks maximize profits by choosing deposit contracts and financial portfolios. They offer a deposit contract (k^r, d^r) to each depositor, which requires the deposit of k^r units of the consumption goods at $t = 1$, in return for a tradeable claim to d^r units of goods at $t = 3$. Besides taking deposits, retail banks borrow ℓ^r from the interbank market at $t = 1$.[10] They also raise equity to finance part of their assets to meet their leverage requirement, which leads to a balance sheet cost because it forces retail bankers to suffer the cost of working to accumulate (sweat) equity (Williamson, 2019). Retail banks invest their funds in m units of reserves and b^r units of government bonds at $t = 1$. Table 1 presents retail banks' balance sheets, or more precisely, the inflow of funds from their asset holdings and payment outflows on their liabilities at $t = 3$.

Competition among retail banks pushes them to offer contracts that maximize depositors' utility. A depositor with a contract (k^r, d^r) then makes a take-it-or-leave-it offer to the producer they meet, exchanging their deposit claims for $c_2 = d^r$ units of consumption

[10] In principle, retail banks could lend to wholesale banks. In such a scenario, wholesale banks would hold government bonds as collateral to secure interbank borrowing. However, because of the risk of wholesale banking failure, retail banks' expected rate of return on such lending must be lower than the rate they directly invest in government bonds. As a result, no retail bank lends to wholesale banks.

Assets	Liabilities and Equity
reserves: $r^m m$	deposit claims: d^r
government bonds: $r^b b^r$	interbank borrowing: $r^\ell \ell^r$
	equity: e

Table 1: Retail Bank's Balance Sheet

goods at $t = 2$, after depositing k^r at $t = 1$.[11] This results in a payoff

$$-k^r + u\left(d^r\right). \tag{1}$$

The limited commitment problem implies that banks' return on assets should exceed their payment on liabilities. As for retail banks, the leverage requirement further restricts their balance sheet in a way such that a fraction θ of their asset returns must cover their liabilities, so[12]

$$\theta \overbrace{\left(r^m m + r^b b^r\right)}^{\text{return on assets}} \geq \underbrace{d^r + r^\ell \ell^r}_{\text{payment on liabilities}}. \tag{2}$$

Finally, a retail bank must make a nonnegative profit to operate, so

$$\overbrace{k^r - d^r}^{\text{return from the deposits}} \underbrace{-m - b^r + \ell^r + r^m m + r^b b^r - r^\ell \ell^r}_{\text{return from financial portfolios}} \geq 0, \tag{3}$$

where $k^r, d^r, m, b^r \geq 0$. In equilibrium, free entry ensures this holds with equality.[13]

To sum up, competitive retail banks choose deposit contract (k^r, d^r) and financial portfolio (m, b^r, ℓ^r) to maximize their depositors' utility (1), subject to the leverage constraint

[11] Under a take-it-or-leave-it offer, depositors offer producers zero surplus from trade. So, the quantity of goods a depositor obtains equals the producer's net payoff from acquiring d^r units of deposit claims.

[12] I impose the leverage requirement at $t = 3$, the stage when banks have to settle their debts, to link this requirement to their limited commitment problem.

[13] From (3) with equality,

$$\underbrace{m + b^r - k^r - \ell^r}_{\text{equity raised at } t=1} = \underbrace{r^m m + r^b b^r - d^r - r^\ell \ell^r}_{\text{return on equity at } t=3},$$

where the left-hand side of the equation denotes the equity raised by retail banks at $t = 1$. This equity must be positive because, from (2), the leverage requirement pushes retail banks to earn a positive equity return at $t = 3$.

(2), nonnegative profit constraint (3) and nonnegative constraints $k^r, d^r, m, b^r \geq 0$. Before solving the retail bank's problem, notice that, except for the nonnegative constraint on government bonds $b^r \geq 0$, other nonnegative constraints never bind in equilibrium. In particular, the nonnegative constraint on reserves $m \geq 0$ does not bind because only retail banks can hold reserves, and their reserve holding will later be determined by the central bank's balance sheet policy. However, constraint $b^r \geq 0$ might bind because government bonds can, in principle, be held by other private agents except retail banks.

The solution to the retail bank's problem gives the following equilibrium conditions

$$r^m \left[1 - \theta + \theta u'\left(d^r\right)\right] = 1, \tag{4}$$

$$r^b \left[1 - \theta + \theta u'\left(d^r\right)\right] + \lambda^r = 1, \tag{5}$$

$$r^\ell u'\left(d^r\right) = 1, \tag{6}$$

where λ^r denotes the Lagrange multiplier for the nonnegative constraint $b^r \geq 0$. Equations (4) and (5) determine retail banks' demands for reserves (m) and government bonds (b^r) associated with their deposit contract. Equation (6) determines the interest rate a retail bank is willing to pay on interbank borrowing.

Wholesale Banks Like retail banks, wholesale banks maximize profits by choosing deposit contracts and financial portfolios. However, a deposit contract in the wholesale banking sector is a triple (k^w, b', d^w), rather than a pair as in the retail banking sector, because wholesale banks must prepare government bonds for depositors' withdrawal requests.[14] Specifically, k^w is the required deposit for each depositor at $t = 1$. In return, each wholesale bank depositor can withdraw b' units of government bonds at the end of $t = 1$ or opt for a tradeable deposit claim to d^w consumption goods at $t = 3$. In this way, wholesale banks provide liquidity insurance to their depositors in the spirit of Dia-

[14]Allowing wholesale banks to hold bonds and deliver them upon withdrawal is harmless and serves as a shortcut for depositors withdrawing funds to buy bonds themselves, especially in my setting, where competitive banks maximize depositor payoffs.

Assets	Liabilities
government bonds: $r^b b^w$	government bonds: $r^b[\rho + (1-\rho)\eta]b'$
interbank lending: $r^\ell \ell^w$	deposit claims: $(1-\rho)(1-\eta)d^w$

Table 2: Wholesale Bank's Balance Sheet

mond and Dybvig (1983). Besides deposits, wholesale banks lend ℓ^w to retail banks and purchase b^w units of government bonds, where part of the bonds are used for depositors' withdrawal requests. In contrast to retail banks, wholesale banks cannot hold reserves. They also do not raise equity as they face no leverage requirement. Table 2 presents wholesale banks' inflows and outflows of funds at $t = 3$.

As with retail banks, wholesale banks maximize depositors' utility to compete for depositors. They diversify across depositors, considering depositors' liquidity shock ρ and withdrawal probability η.[15] The expected utility of a wholesale bank depositor is

$$-k^w + [\rho + (1-\rho)\,\eta]\, u\left(r^b b'\right) + (1-\rho)\,(1-\eta)\,\delta u\,(d^w)\,. \tag{7}$$

That is, after paying the required deposit k^w, a fraction $\rho + (1-\rho)\,\eta$ of the wholesale depositors withdraw government bonds and make a take-it-or-leave-it offer that exchanges for $r^b b'$ units of consumption goods from the producer they meet. The remaining $(1-\rho)\,(1-\eta)$ of depositors, who choose not to withdraw, successfully trade their deposit claims for d^w units of goods with probability δ. By contrast, the rest $1-\delta$ of them, whose banks default, lose their claims and receive zero payoff (recall $u(0) = 0$).

Wholesale banks are not subject to the leverage requirement. However, they must prepare assets that satisfy the following collateral constraint at $t = 1$,

$$\overbrace{r^b\,[b^w - [\rho + (1-\rho)\,\eta]\,b'] + r^\ell \ell^w}^{\text{return on assets}} \;\geq\; \underbrace{(1-\rho)\,(1-\eta)\,d^w}_{\text{payment on liability}}, \tag{8}$$

[15]Banks serve many depositors, while each depositor can contact only one bank. Although depositors cannot diversify across banks, they can observe all banks' contracts and choose the optimal one.

to show their willingness to pay off their debts (unless they fail). This constraint holds for all wholesale banks ex-ante. But, again, a fraction $1-\delta$ of wholesale banks will fail at $t=2$, and this potential banking failure triggers depositors' panicking behavior reflected by the endogenous withdrawal probability η.

Finally, wholesale banks also require a nonnegative profit to operate,

$$\overbrace{k^w - (1-\rho)(1-\eta)\delta d^w}^{\text{return from deposits}} \underbrace{-b^w - \ell^w + r^b \left[b^w - \left[\rho + (1-\rho)\eta\right] b'\right] + r^\ell \ell^w}_{\text{return from financial portfolio}} \;\geq\; 0, \tag{9}$$

where $k^w, b', d^w, b^w, b^w - \left[\rho + (1-\rho)\eta\right] b' \geq 0$. Wholesale banks only pay off their deposit claims if they do not fail, which occurs with probability δ. Otherwise, they abscond with assets and consume the asset returns. Wholesale banks have some of their government bonds withdrawn by depositors, meaning they only earn returns on the remaining part, i.e., $b^w - \left[\rho + (1-\rho)\eta\right] b'$. Again, (9) holds with equality under free entry.

To sum up, wholesale banks choose deposit contract (k^w, b', d^w) and financial portfolio (b^w, ℓ^w) to maximize a representative wholesale depositor's expected utility (7), subject to the collateral constraint (8), nonnegative profit constraint (9) and nonnegative constraints $k^w, b', d^w, b^w, b^w - \left[\rho + (1-\rho)\eta\right] b' \geq 0$. As with the retail bank's problem, notice that, except the nonnegative constraint on government bonds $b^w - \left[\rho + (1-\rho)\eta\right] b' \geq 0$, other nonnegative constraints never bind in equilibrium.

The solution to the wholesale bank's problem gives the following equilibrium conditions

$$r^b u'\left(r^b b'\right) \;=\; 1, \tag{10}$$

$$r^b \left[1 - \delta + \delta u'\left(d^w\right)\right] + \lambda^w \;=\; 1, \tag{11}$$

$$r^\ell \left[1 - \delta + \delta u'\left(d^w\right)\right] \;=\; 1, \tag{12}$$

where λ^w denotes the Lagrange multiplier for $b^w - \left[\rho + (1-\rho)\eta\right] b' \geq 0$. Equation (10) determines wholesale banks' demand for government bonds associated with each depositor's withdrawal request (i.e., $\left[\rho + (1-\rho)\eta\right] b'$), while equation (11) determines their demand for government bonds to back their deposit claims (i.e., $b^w - [\rho + (1-\rho)\eta]\, b'$). Wholesale banks can also use claims on interbank lending to back their deposit claims, and equation (12) determines such demand.

Remark The equilibrium conditions displayed above and those in the retail bank's problem are asset pricing kernels in consumption-based capital asset pricing models, surveyed by Campbell (1999). For example, the government bond price, i.e., the inverse of the gross real interest rate on bonds $1/r^b$ in (10), is equal to wholesale bank depositors' marginal return of trading with bonds. Other conditions, (4), (5), (6), (11) and (12), have similar interpretations by properly adjusting parameters related to the leverage requirement θ, banking failure probability $1-\delta$, and binding constraints λ^r and λ^w.

2.2 Flight-to-Safety by Wholesale Bank Depositors

The endogenous withdrawal probability η captures the severity of wholesale banking panics or the magnitude of flight-to-safety. Depositors' expected payoffs determine their withdrawal probability and give rise to three scenarios for wholesale banking panics:

$$\text{(i) No banking panic:} \qquad \eta = 0, \qquad \text{if } u\left(r^b b'\right) \leq \delta u\left(d^w\right); \tag{13}$$

$$\text{(ii) Partial banking panic:} \qquad 0 < \eta < 1, \qquad \text{if } u\left(r^b b'\right) = \delta u\left(d^w\right); \tag{14}$$

$$\text{(iii) Full banking panic:} \qquad \eta = 1, \qquad \text{if } u\left(r^b b'\right) \geq \delta u\left(d^w\right). \tag{15}$$

From (13)-(15), a no banking panic equilibrium occurs if wholesale bank depositors prefer wholesale banks' deposit claims to government bonds, a partial banking panic occurs if these depositors are indifferent between these two options, and a full banking panic occurs if government bonds are preferred.

2.3 Government

At the beginning of period 1, the fiscal authority issues $\hat{b}$ units of government bonds and transfers the revenue τ_1 to depositors:

$$\hat{b} = \tau_1. \tag{16}$$

The central bank conducts an asset swap, purchasing $\hat{b} - \bar{b}$ units of government bonds with reserves. So,

$$\underbrace{\hat{b} - \bar{b}}_{\text{asset}} = \underbrace{\bar{m}}_{\text{liability}}, \tag{17}$$

where $\bar{m}$ and $\bar{b}$ represent the amounts of reserves and government bonds circulating within the private sector, respectively. Fiscal policy determines the total supply of government bonds ($\hat{b} = \bar{m} + \bar{b}$). The central bank adjusts the relative supply of government bonds and reserves in the private sector through open market operations. The supply of reserves, $\bar{m}$, describes the central bank balance sheet policy, representing the size of its balance sheet. I focus on the effects of this balance sheet policy while holding fiscal policy constant.

Government bonds and central bank reserves are redeemed in period 3. The fiscal authority taxes depositors τ_3 lump sum to payoff its debts and transfers τ^{cb} (receives, if negative) to the central bank to support its payments:

$$r^b\hat{b} + \tau^{cb} = \tau_3. \tag{18}$$

The central bank pays off its reserves, using the returns from its holdings of government bonds and the transfer from the fiscal authority:

$$r^m\bar{m} = r^b\left(\hat{b} - \bar{b}\right) + \tau^{cb}. \tag{19}$$

2.4 Definition of Equilibrium

I focus on equilibria in which retail banks' leverage and wholesale banks' collateral constraints bind. As a result, banks cannot provide deposit claims that allow depositors to consume the satiation level, i.e., $c_2 < c^\star$ with $u'(c^\star) = 1$. Otherwise, depositors would always consume $c^\star$, and there would be no change in allocation in response to central bank interventions. To ensure these constraints bind, I assume a scarcity of total government bond supply following Andolfatto and Williamson (2015) and Williamson (2019). Specifically, Assumption 1 states that retail banks cannot support the satiated consumption level for their own depositors, even after exhausting all government bonds supplied by the fiscal authority.

Assumption 1 (Scarcity of Total Government Bond Supply)**.** *Total supply of government bonds is scarce, such that $\theta\hat{b} < \alpha c^\star$ with $u'(c^\star) = 1$.*

Definition 1 (Equilibrium)**.** *Given fiscal policy $\hat{b}$ and central bank balance sheet policy $\bar{m}$, an equilibrium consists of an allocation $\left(\bar{b}, d^r, m, b^r, \ell^r, b', d^w, b^w, \ell^w, c_2^r, c_2^b, c_2^w\right)$, Lagrange multipliers λ^r and λ^w, market-determined real interest rates (r^m, r^ℓ, r^b), and a withdrawal probability η, satisfying the binding leverage constraint (2) and binding collateral constraint (8), equilibrium conditions for private banks' problems (4)-(12), one of conditions (13)-(15) to support wholesale bank depositors' withdrawal strategy, market clearing,*

$$\alpha m = \bar{m} \qquad \textit{(reserve market)}, \tag{20}$$

$$\alpha b^r + (1-\alpha)\, b^w = \bar{b} \qquad \textit{(government bond market)}, \tag{21}$$

$$\alpha \ell^r = (1-\alpha)\, \ell^w \qquad \textit{(interbank market)}, \tag{22}$$

and the complementary-slackness conditions with corresponding nonnegative constraints,

$$\lambda^r b^r = 0, \quad \lambda^r \geq 0, \quad b^r \geq 0, \tag{23}$$

$$\lambda^w [b^w - [\rho + (1-\rho)\,\eta]\, b'] = 0, \quad \lambda^w \geq 0, \quad b^w - [\rho + (1-\rho)\,\eta]\, b' \geq 0, \tag{24}$$

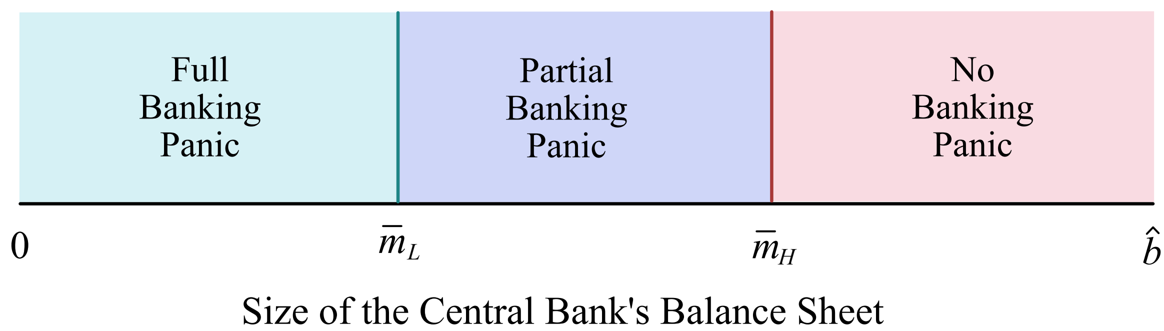

Figure 2: How To Determine the Type of Equilibrium

where $c_2^r = d^r$, $c_2^b = r^b b'$, and $c_2^w = d^w$ are consumption quantities for retail bank depositors, wholesale bank depositors trade with government bonds, and wholesale bank depositors trade with deposit claims.

3 Equilibrium

Depending on parameter values, there will be three types of equilibrium: no banking panic ($\eta = 0$), partial banking panic ($0 < \eta < 1$), and full banking panic ($\eta = 1$). For each type, I examine the effects of a central bank balance sheet expansion, implemented through purchases of government bonds from the private sector by issuing new reserves. As will become clear throughout this section, this policy alters the relative attractiveness of wholesale bank deposits versus safe government bonds, shifting depositors' asset demand and, in turn, affecting the severity of banking panics or the magnitude of flight-to-safety.

As illustrated in Figure 2, expanding the size of the central bank's balance sheet reduces the severity of wholesale banking panics. Proposition 1 formalizes the result, and I prove it in a constructive way by first providing conditions that solve a partial banking panic equilibrium and then establishing the role of the size of the central bank's balance sheet, $\bar{m}$, in sustaining different equilibria.[16] I will discuss the intuition in detail in the partial banking panic scenario because that is the only scenario where the withdrawal

[16]I present all proofs in the Appendix A and discuss the intuition in the main text.

probability, η, changes in response to central bank interventions.

Proposition 1. *Under Assumption 1, there are two thresholds for the size of the central bank's balance sheet with* $0 < \bar{m}_L < \bar{m}_H < \hat{b}$*, where* $\bar{m}_L$ *and* $\bar{m}_H$ *solve the system of equations* (14)*,* (25)*,* (26)*, and* (27) *with* $\eta = 1$ *and* $\eta = 0$*, respectively. The size of the central bank's balance sheet determines the equilibrium as follows:*

1. *a full banking panic,* $\eta = 1$*, occurs when the size of the central bank's balance sheet is below the lower threshold such that* $\bar{m} \in (0, \bar{m}_L]$*;*

2. *a partial banking panic,* $\eta \in (0, 1)$*, occurs when* $\bar{m} \in (\bar{m}_L, \bar{m}_H)$*;*

3. *no banking panic,* $\eta = 0$*, occurs when* $\bar{m} \in [\bar{m}_H, \hat{b})$*.*

Moreover, these thresholds increase with the probability of wholesale banking failure $1 - \delta$*, i.e.,* $\frac{\partial \bar{m}_L}{\partial \delta} < 0$ *and* $\frac{\partial \bar{m}_H}{\partial \delta} < 0$*.*

In addition to the equilibrium type determination, Proposition 1 shows that if wholesale banking becomes more risky, the central bank can prevent additional withdrawals by expanding its balance sheet. Specifically, swapping government bonds for reserves makes deposit claims more attractive to depositors, helping offset the increased risk of banking.

3.1 The Role of Government Bonds

Before delving into the details, I establish two important intermediate results (Lemmas 1 and 2), showing that no bank holds government bonds as collateral in any type of equilibrium. These results hold under Assumption 1 that restricts the fiscal policy, but are independent of monetary policy.

Lemma 1. *Retail banks never hold government bonds, i.e.,* $b^r = 0$ *and* $\lambda^r > 0$*.*

Retail banks bear a balance sheet cost of holding assets because they must raise sweat equity to finance their investment. Therefore, they hold a positive stock of government bonds only when the interest on bonds is strictly higher than the interest rate they pay for interbank borrowing. However, wholesale banks always ask for a higher interest rate on interbank lending than on government bonds, as bonds are always available to them. As a result, retail banks never hold government bonds in equilibrium.

Lemma 2. *Wholesale banks only purchase government bonds for depositors' withdrawal requests, i.e.,* $b^w - [\rho + (1-\rho)\eta]\, b' = 0$.

Wholesale bank depositors prefer directly using government bonds in exchange rather than letting banks hold bonds as collateral to back tradeable deposit claims. Otherwise, there is a chance that their bank will fail and abscond. To entertain depositors' preferences, competitive wholesale banks assign all the bonds for depositors' withdrawal requests, basically dividing their bond holdings evenly across the depositors they serve.

Remark Under Lemma 1, conditions (4), (5), (11), and (12) imply that the interest rate on reserves is higher than the interest rate on interbank loans, which itself is higher than the interest rate on government bonds, i.e., $r^m > r^\ell \geq r^b$. This interest rate structure is consistent with the short-term interest rate structure in the U.S. since the Federal Reserve started paying interest on reserve balances in October 2008.[17] Canada also exhibits similar features. Intuitively, reserves are traded at a low price and a high real interest rate as they can only circulate among retail banks. By contrast, government bonds are traded at a high price and a low real interest rate as everyone, including depositors and producers, can hold them. The interest rate on interbank loans lies between the above two rates because retail and wholesale banks can hold these loans, but not other private agents.

[17]In the U.S., the interest rate on reserve balances is higher than the federal funds rate, which is higher than short-term T-bill rates. Although the federal funds rate is the overnight lending rate among depository institutions (i.e., retail banks) and government-sponsored enterprises (GSEs), it is a good proxy of the interbank rate in this paper. The reason is that, in the last decade, most of the federal fund market activities involve lending from GSEs, which cannot directly obtain the yields on reserves, to depository institutions. Moreover, GSEs are not subject to stringent regulations like depository institutions, suffering few balance sheet costs. These features are the same as those developed in this paper.

3.2 Partial Banking Panic

I begin with the partial banking panic ($0 < \eta < 1$) equilibrium under $\bar{m} \in (\bar{m}_L, \bar{m}_H)$. In what follows, I express equilibrium conditions in terms of depositors' withdrawal probability η and consumption quantities (c_2^r, c_2^b, c_2^w), as the center of the analysis is on changes in withdrawal behavior and welfare in response to central bank interventions. The depositors' consumption level reflects the trading surplus between depositors and producers at $t = 2$, which determines welfare in this model, given that producers and bankers earn zero profits in equilibrium.[18] Under Assumption 1, depositors consume below their satiation level, so any increase (decrease) in consumption raises (reduces) welfare.

Existence and Uniqueness of Equilibrium In this equilibrium, wholesale bank depositors are indifferent between government bonds and deposit claims as in (14). The equilibrium is the unique solution to a system of equations consisting of condition (14), along with the no-arbitrage condition (25), collateral market clearing condition (26), and bond market clearing condition (27) explained below.

No-arbitrage Interbank Market: From asset pricing kernels (6) and (12), retail banks are willing to pay a real interest rate $1/u'(c_2^r)$ on their interbank borrowing, while wholesale banks ask for a real interest rate $1/(1 - \delta + \delta u'(c_2^w))$ on their interbank lending. Therefore, the following no-arbitrage condition must hold in equilibrium:

$$u'(c_2^r) \; = \; 1 - \delta + \delta u'(c_2^w) \, . \tag{25}$$

As depicted in Figure 3, this no-arbitrage condition gives an upward-sloping curve in the c_2^r–c_2^w space because of the decreasing marginal utility in consumption.

Collateral Market Clearing: The binding leverage constraint (2) and collateral constraint (8), market clearing conditions (20) and (22), condition (4) and no-arbitrage condition

[18] Recall that producers make zero profit because depositors extract the entire trading surplus through take-it-or-leave-it offers, and bankers earn zero profit due to free entry.

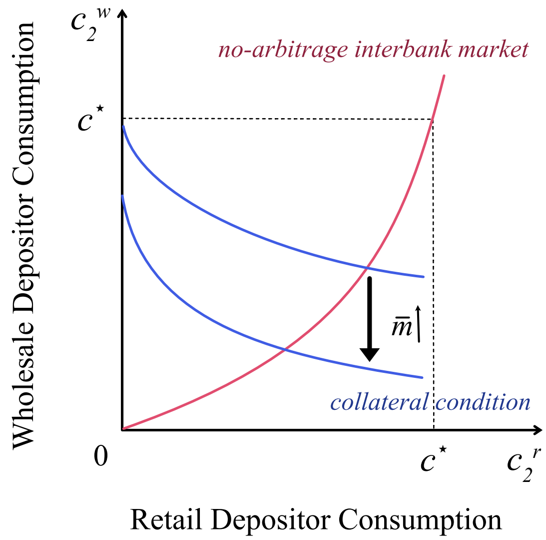

Figure 3: Partial Banking Panic Equilibrium

(25) give the following equilibrium collateral market clearing condition:

$$\underbrace{\theta\bar{m}}_{\text{effective collateral supply (i.e., reserves)}} = \underbrace{\alpha c_2^r \left[1-\theta+\theta u'\left(c_2^r\right)\right]}_{\text{retail banks' demand for collateral}} + \underbrace{(1-\alpha)(1-\rho)(1-\eta)\, c_2^w \left[1-\theta\delta+\theta\delta u'\left(c_2^w\right)\right]}_{\text{wholesale banks' demand for collateral}}. \tag{26}$$

This condition equates the effective collateral supply to private banks' demand.[19] Reserves work as collateral backing all private banks' deposit claims, as no bank holds government bonds as collateral (Lemmas 1 and 2). Although reserves are inaccessible to wholesale banks, they back these banks' deposit claims indirectly through the interbank market.

Crucially, the severity of wholesale banking panics determines the aggregate demand for collateral because the withdrawal probability η determines the fraction of depositors who demand wholesale banks' deposit claims ultimately backed by reserves. For example, a decrease in η shifts this demand rightward.

<u>Bond Market Clearing</u>: The fiscal policy rule $\hat{b} = \bar{m} + \bar{b}$, equilibrium condition (10), market clearing condition (21), and binding nonnegative constraint $b^w - [\rho + (1-\rho)\eta]\, b' = 0$ give

[19] Retail banks demand collateral for two reasons: to secure deposit claims or interbank loans. Here, the notion of "retail banks' demand for collateral" means the demand for deposit claims only because wholesale banks further use interbank loans to back their deposit claims.

the following equilibrium bond market clearing condition:

$$\overbrace{\hat{b} - \bar{m}}^{\text{government bond supply}} = \underbrace{(1-\alpha)\left[\rho + (1-\rho)\eta\right] c_2^b u'\left(c_2^b\right)}_{\text{aggregate demand for government bonds}}. \tag{27}$$

The left-hand side represents the government bond supply in the private sector, which is the fiscal authority's total government bond supply minus the bonds held by the central bank to back its reserves. The right-hand side represents the private sector's aggregate demand for bonds, where wholesale bank depositors use all outstanding government bonds to settle their transactions because, again, no bank holds bonds as collateral.

As with the collateral market clearing condition, the severity of wholesale banking panics determines the aggregate demand for government bonds. For example, a decrease in η shifts this demand leftward.

The conditions (14), (26), and (27) mentioned above implicitly define a downward-sloping *collateral condition* in the c_2^r–c_2^w space as depicted in Figure 3.[20] Reserves serve as the ultimate collateral backing both retail and wholesale bank deposits, which depositors use to exchange for consumption goods. The downward slope reflects that, with a fixed reserve supply, an increase in one type of exchange crowds out the other.

Graphically, the collateral condition and the upward-sloping no-arbitrage condition determine a unique equilibrium characterized by the consumption quantities c_2^r and c_2^w. These consumption quantities do not exceed the satiated level $c^\star$ under Assumption 1 because the assumption regarding the scarcity of total government bond supply constrains the collateral condition to be close to the original point. I can then solve for other equilibrium outcomes based on c_2^r and c_2^w. For example, (26) solves for the withdrawal probability η. Proposition 2 below summarizes these results.

Proposition 2 (Unique Partial Banking Panic Equilibrium)**.** *Under Assumption 1 and*

[20] See Appendix A.1 for the detailed proof. The label "collateral condition" is a slight abuse of language. I use this label because the collateral market clearing condition explains the comparative statics I study when $\bar{m}$ changes.

$\bar{m} \in (\bar{m}_L, \bar{m}_H)$*, there exists a unique partial banking panic equilibrium characterized by the consumption allocation* $c_2^r, c_2^w, c_2^b \in (0, c^\star)$ *and the withdrawal probability* $\eta \in (0,1)$*.*

Central Bank Balance Sheet Expansion I evaluate the effects of an expansion in the size of the central bank's balance sheets ($\bar{m}$) by performing comparative statics on the system of equations (14), (25), (26), and (27).

Proposition 3 (Effects of Balance Sheet Expansions)**.** *Assume* $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ *with* $0 < \sigma < 1$*, an expansion of the central bank's balance sheet mitigates wholesale banking panics, i.e.,* $\frac{\partial \eta}{\partial \bar{m}} < 0$*, but impedes transactions and harms depositors, i.e.,* $\frac{\partial c_2^b}{\partial \bar{m}}, \frac{\partial c_2^r}{\partial \bar{m}}, \frac{\partial c_2^w}{\partial \bar{m}} < 0$*.*

The CRRA function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ satisfies earlier assumptions for utility function. In particular, $0 < \sigma < 1$ is a necessary condition for $-c\frac{u''(c)}{u'(c)} < 1$, which implies that the substitution effect dominates the income effect. Consequently, the demand for assets increases with their rate of return.

Corollary 1 (Effects on Real Interest Rates)**.** *An expansion in the size of the central bank's balance sheet reduces the real interest rates on government bonds, reserves, and interbank loans, i.e.,* $\frac{\partial r^b}{\partial \bar{m}}, \frac{\partial r^m}{\partial \bar{m}}, \frac{\partial r^\ell}{\partial \bar{m}} < 0$*.*

The interest rate movements in Corollary 1 are consistent with the empirical findings of Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2020), who show that the central bank balance sheet expansion, through quantitative easing, lowers both interest rates on government bonds and short-term interest rates such as repo rates. As it will be clear in a moment, their empirical evidence also supports the key mechanism that generates the general equilibrium effects in this paper: by purchasing bonds, the central bank reduces the net supply of bonds available to the private market, increasing their scarcity and driving down their interest rates.

<u>Flight-to-Safety Implication</u>: An expansion in the size of the central bank's balance sheet has three effects. *First*, this expansion increases the effective collateral supply by issu-

ing more reserves, which relaxes retail banks' leverage constraint and wholesale banks' collateral constraint, promoting the supply of bank liabilities. In particular, it facilitates the supply of wholesale banks' deposit claims, putting upward pressure on their interest rate. *Second*, this expansion reduces the supply of government bonds in the private sector, lowering their interest rate and making them a less attractive safe harbor for wholesale bank depositors. The first two direct partial equilibrium effects imply the *third* effect: wholesale bank depositors' withdrawal probability decreases (i.e., $\frac{\partial \eta}{\partial \bar{m}} < 0$), mitigating the severity of wholesale banking panics, as deposit claims become more attractive relative to government bonds.

The flight-to-safety implication described above under a partial banking panic equilibrium can be extended to the general case when characterizing the equilibrium type. As summarized in Proposition 1, central bank balance sheet expansions gradually reduce the severity of panics, starting from full banking panics ($\eta = 1$) to eventually eliminate them entirely ($\eta = 0$).

Welfare Implication: Despite mitigating wholesale banking panics, expanding the central bank's balance sheet reduces welfare. This striking result follows from the interaction of the three effects mentioned above, which jointly determine the change in the consumption allocation $\left(c_2^r, c_2^w, c_2^b\right)$. A central bank balance sheet expansion puts pressure on the collateral condition in Figure 3 to shift upward by increasing the effective collateral supply as in (26). However, the withdrawal probability η decreases in response to this expansion, increasing the demand for collateral and, therefore, shifting the collateral condition downward. I show that the latter effect necessarily dominates the former one. Collateral becomes relatively more scarce, harming depositors who trade with deposit claims backed by collateral. Depositors who trade with government bonds also get worse by (14). Clearly, the endogenous shift in asset demand is crucial for these results. Different welfare implications arise when there is no such shift in response to the balance sheet expansion, as in the scenarios below with no banking panic or full banking panic.

The reduction in depositors' transactions, and thus in welfare, can be understood more intuitively through individual decisions. *Firstly*, an expansion in the central bank's balance sheet harms depositors who trade with government bonds. Expanding the balance sheet reduces the government bond supply in the private sector as in (27), putting pressure on the interest rate on government bonds to fall. Although the decreased demand for bonds that comes from a decrease in withdrawal probability η puts pressure on the bond interest rate to rise, the decreased supply of bonds dominates this decreased demand. This implies a reduction in the interest rate on government bonds (i.e., $\frac{\partial r^b}{\partial \bar{m}} < 0$), thereby a lower trading volume for transactions settled with these safe assets (i.e., $\frac{\partial c_2^b}{\partial \bar{m}} < 0$).

Secondly, an expansion in the central bank's balance sheet harms depositors who trade with wholesale banks' deposit claims. The increased reserve supply increases the effective collateral supply as in (26), relaxing retail banks' leverage constraint and, through the interbank market, relaxing wholesale banks' collateral constraint. Consequently, there is an increase in wholesale banks' supply of deposit claims. However, on the other side of the market, the reduction in the withdrawal probability η increases the demand for these claims. The increased demand for deposit claims dominates their increased supply. Each depositor obtains fewer claims in exchange, thereby, a lower trading volume for transactions settled with these claims (i.e., $\frac{\partial c_2^w}{\partial \bar{m}} < 0$). Furthermore, this increased demand for deposit claims intensifies banks' competition for collateral. Wholesale banks compete for interbank loans, asking for a lower interest rate on them (i.e., $\frac{\partial r^\ell}{\partial \bar{m}} < 0$). Retail banks compete for reserves to back their interbank borrowing, asking for a lower real rate of return on reserves (i.e., $\frac{\partial r^m}{\partial \bar{m}} < 0$).

Finally, an expansion in the central bank's balance sheet harms depositors who trade with retail banks' deposit claims. Unlike wholesale banks, the increased reserve supply does not increase the supply of retail banks' deposit claims, even if it relaxes their leverage constraint, which works like an income effect. That's because, besides depositors, retail banks raise funds from wholesale banks, who ask for a lower rate of return r^ℓ in response to the expansion (recall that $\frac{\partial r^\ell}{\partial \bar{m}} < 0$). They substitute their funding source for cheaper interbank borrowing. The substitution effect from a decrease in r^ℓ dominates the income effect from relaxing the leverage constraint. As a result, retail banks reduce their supply of deposit claims, implying a lower trading volume for transactions settled with these claims (i.e., $\frac{\partial c_2^r}{\partial \bar{m}} < 0$).

Damaging Effects of Central Bank Balance Sheet Expansions In the above scenario, the decrease in the supply of government bonds dominates the decrease in their demand, while the increase in the demand for wholesale deposits dominates the increase in their supply or, more broadly, the increased supply of the underlying collateral that supports these deposits. These results arise because central bank balance sheet expansions take out government bonds from the private sector, ultimately reducing welfare. In particular, the decreased government bond supply reduces their interest rate, playing an indirect but significant role in the markets for wholesale deposits and collateral. As bonds become less attractive, depositors switch to wholesale bank deposits backed by collateral, contributing to the dominance effects in the markets for these assets. In section 4, I demonstrate how these results can be reversed by implementing a central bank policy that does not change the government bond supply. This alternative policy also serves as a counterfactual for isolating the role of the government bond market.

3.3 No Banking Panic and Full Banking Panic

The effects of an expansion in the size of the central bank's balance sheet are qualitatively similar in a no banking panic ($\eta = 0$) and a full banking panic ($\eta = 1$) equilibrium because the endogenous withdrawal probability, η, remains constant in either case. As in the partial banking panic equilibrium, the balance sheet expansion harms depositors who trade with government bonds. However, it now benefits depositors who trade with retail and wholesale banks' deposit claims. That is, the central bank cannot improve welfare, in the Pareto sense, by adjusting the size of its balance sheet.

No Banking Panic ($\eta = 0$ under $\bar{m} \in [\bar{m}_H, \hat{b})$) The market structure is similar to the one studied before, except that no wholesale bank depositor withdraws for safety concerns. Wholesale banks make sufficient revenues from interbank lending, allowing them to offer attractive deposit claims that prevent a wholesale banking panic from arising.

The no-arbitrage condition holds the same as before, while the collateral and bond market clearing conditions become

$$\theta\bar{m} = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] + (1-\alpha)(1-\rho)\, c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right], \tag{28}$$

$$\hat{b} - \bar{m} = (1-\alpha)\,\rho c_2^b u'\left(c_2^b\right), \tag{29}$$

respectively, by setting $\eta = 0$ in (26) and (27).

The no-arbitrage condition (25) and the collateral market clearing condition (28) jointly determine the consumption quantities (c_2^r, c_2^w) for depositors who trade with retail and wholesale banks' deposit claims. The bond market clearing condition (29) solely determines the consumption quantity c_2^b for depositors who trade with government bonds. Under Assumption 1, these conditions solve a unique equilibrium with $0 < c_2^r, c_2^w, c_2^b < c^\star$.

An expansion in the central bank's balance sheet benefits depositors who trade with retail and wholesale banks' deposit claims by relaxing retail banks' leverage and wholesale banks' collateral constraints. First of all, this expansion increases the supply of wholesale banks' deposit claims, which are ultimately backed by reserves. This increased supply implies a higher trading volume for depositors who trade with these claims (i.e., a higher c_2^w) because, unlike a partial banking panic equilibrium, there is no shift in demand for these claims. Furthermore, the absence of the shift in demand puts no pressure on wholesale banks to compete for interbank loans as collateral. The interest rate on interbank loans increases (i.e., a higher r^ℓ), and retail banks have little incentive to substitute their funding source from deposits to interbank loans. With more slack leverage constraints, retail banks increase their supply of deposit claims, implying a higher trading volume for their depositors (i.e., a higher c_2^r).[21]

The central bank balance sheet expansion, however, harms depositors who trade with government bonds. The supply of government bonds in the private sector decreases with an increased reserve supply, while the aggregate demand for bonds remains unchanged as depositors' withdrawal probability remains constant. As in a partial banking panic equilibrium, the decreased supply of government bonds dominates their change in demand, implying a reduction in the interest rate on bonds (i.e., a lower r^b) and a lower trading volume for depositors who demand them (i.e., a lower c_2^b).

Full Banking Panic ($\eta = 1$ under $\bar{m} \in (0, \bar{m}_L]$) The interbank market becomes inactive in a full banking panic equilibrium because wholesale banks do not demand interbank loans to secure deposits. One interpretation of this equilibrium is that wholesale banks resemble narrow banks in that they only purchase safe government bonds for depositors' withdrawal requests. On the other hand, the equilibrium outcomes would be the same if depositors were allowed to invest in government bonds themselves, as competitive banks maximize their depositors' utility under free entry. Therefore, this full banking panic equilibrium also characterizes the feature of equilibrium with disintermediation in the wholesale banking sector.

When the withdrawal probability $\eta = 1$, only retail banks require collateral to back their deposit claims, and the collateral market clearing condition becomes

$$\theta\bar{m} \; = \; \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{30}$$

[21] In this equilibrium, evaluating the effects of an expansion in the size of the central bank's balance sheet on depositors who trade with deposit claims is performing comparative statics on the system of equations (25) and (28) with respect to an increase in reserve supply $\bar{m}$. In what follows, the effects of this expansion on depositors who trade with government bonds are determined by (29).

which is retail banks' leverage constraint. The bond market clearing condition becomes

$$\hat{b} - \bar{m} \; = \; (1-\alpha)\, c_2^b u' \left(c_2^b\right). \tag{31}$$

Conditions (30) and (31) determine the consumption allocation for retail and wholesale bank depositors, respectively. A central bank balance sheet expansion has similar effects to its no banking panic counterpart: it benefits depositors who trade with deposit claims while harming those who trade with government bonds.

4 Expanding the Reach of Central Bank Liabilities

I now study what happens if the central bank expands the reach of its interest-bearing reserves to wholesale banks. There are two reasons for doing this. Firstly, and most importantly, I demonstrate that this enables the central bank to mitigate wholesale banking panics and promote transactions, which in turn lead to welfare improvements. The central bank cannot achieve all of these objectives by adjusting the size of its balance sheet. Secondly, and of theoretical interest, this extension provides a counterfactual to the previous case. Mainly, I show that the increased supply of wholesale bank deposits can dominate their increased demand when the central bank does not withdraw government bonds from the private sector, implying an increase (instead of a decrease) in asset returns. This highlights, again, the critical role of the government bond market in evaluating central bank balance sheet policies (see Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2020) and the references therein).

Specifically, I introduce another form of central bank liabilities in addition to reserves that functions like the U.S. Federal Reserve's overnight reverse repurchase agreement facility (ON-RRP). I call them ON-RRPs (o) for convenience. Both retail and wholesale banks can hold ON-RRPs, so introducing these liabilities works like expanding the reach of reserve accounts to wholesale banks. Unlike government bonds, ON-RRPs can only circulate among financial institutions, and depositors cannot use them directly to settle transactions. This implies that, in equilibrium, the real interest rate on ON-RRPs equals the real interest rate on interbank loans with $r^o = r^\ell$.

The central bank's balance sheet policy now has two dimensions. Firstly, the central bank determines the size of its balance sheet, now given by $s = \bar{m} + \bar{o}$. An expansion in s has the same effects as before when only reserves are available. Secondly, the central bank determines the composition of its liabilities, captured by the supply of ON-RRPs $\bar{o}$. I focus on the effects of a swap of reserves ($\bar{m}$) for ON-RRPs ($\bar{o}$) while holding the size of the central bank's balance sheet, s, constant.[22]

Consider the scenario of a partial banking panic with an active interbank market. Retail banks invest in reserves (m) and borrow ℓ^r from wholesale banks. They do not hold ON-RRPs, for the same reason they do not hold government bonds as explained in Lemma 1. Instead, wholesale banks hold these ON-RRPs (o). They also lend ℓ^w to retail banks and hold government bonds (b^w) for their depositors' withdrawal requests. Table 3 presents private banks' inflows and outflows of funds at $t = 3$.

A swap of reserves for ON-RRPs increases the supply of ON-RRPs that are directly accessible to wholesale banks. In response, wholesale banks reduce their lending to retail banks to gain indirect access to reserves. In this way, this swap boosts the effective collateral supply by avoiding the inefficiency associated with retail banks' leverage requirements. To see this, notice that now the collateral market clearing condition becomes

$$\begin{aligned} \theta s + (1-\theta)\,\bar{o} \; &= \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right] \\ &\quad + (1-\alpha)(1-\rho)(1-\eta)\, c_2^w \left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right]. \end{aligned} \tag{32}$$

From (32), an increase in the supply of ON-RRPs $\bar{o}$ increases the effective collateral supply that is captured by the left-hand side of the equation. This effective collateral supply can

[22]In practice, the mix between ON-RRPs and reserves is determined by the interest rates set by the central bank. This swap can be viewed as a convenient shortcut to capturing such interest rate policy, and one can obtain the same results considering an experiment that fixes s while reducing the interest rate spread between reserves and ON-RRPs, i.e., reducing r^m/r^o.

(a) Retail Bank

Assets	Liabilities and Equity
reserves: $r^m m$	deposit claims: d^r
	interbank borrowing: $r^\ell \ell^r$
	bank capital: e

(b) Wholesale Bank

Assets	Liabilities
government bonds: $r^b b^w$	government bonds: $r^b[\rho + (1-\rho)\eta]b'$
interbank lending: $r^\ell \ell^w$	deposit claims: $(1-\rho)(1-\eta)d^w$
ON-RRPs: $r^o o$	

Table 3: Private Banks' Balance Sheets with Overnight Reverse Repurchase Agreements

also be written as $\theta\bar{m} + \bar{o}$ to reflect the fact that assets held by retail banks can secure fewer liabilities than those held by wholesale banks.

The bond market clearing condition (27) becomes

$$\hat{b} - s \;=\; (1-\alpha)\left[\rho + (1-\rho)\,\eta\right] c_2^b u'\left(c_2^b\right), \tag{33}$$

where the supply of government bonds in the private sector only depends on the size of the central bank's balance sheet. Therefore, the bond supply does not change when the central bank swaps between reserves and ON-RRPs.

Swapping Reserves for ON-RRPs Evaluating the effects of a swap of reserves for ON-RRPs is performing comparative statics on the system of equations (14), (25), (32), and (33).

Proposition 4. *A swap of reserves for ON-RRPs mitigates wholesale banking panics, facilitates transactions and benefits depositors, i.e.,* $\frac{\partial\eta}{\partial\bar{o}} < 0$ *and* $\frac{\partial c_2^r}{\partial\bar{o}}, \frac{\partial c_2^w}{\partial\bar{o}}, \frac{\partial c_2^b}{\partial\bar{o}} > 0$.

Corollary 2. *A swap of reserves for ON-RRPs increases the real interest rates on government bonds, reserves, interbank loans, and ON-RRPs, i.e.,* $\frac{\partial r^b}{\partial\bar{o}}, \frac{\partial r^m}{\partial\bar{o}}, \frac{\partial r^\ell}{\partial\bar{o}}, \frac{\partial r^o}{\partial\bar{o}} > 0$.

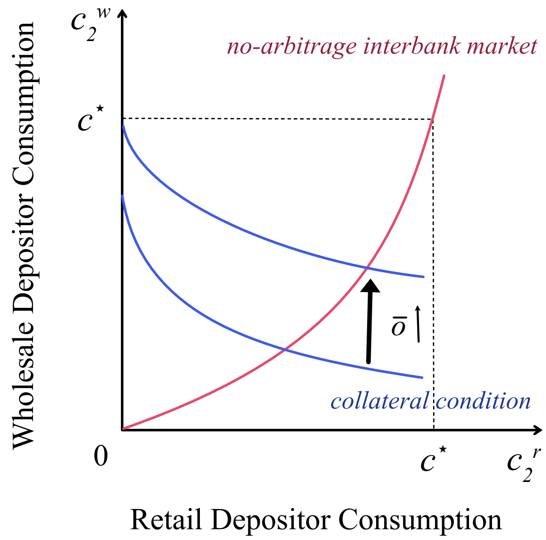

Figure 4: Partial Banking Panic Equilibrium with ON-RRPs

Like central bank balance sheet expansions, a swap of reserves for ON-RRPs mitigates wholesale banking panics (i.e., $\frac{\partial \eta}{\partial \bar{o}} < 0$). As in (32), this swap increases the effective collateral supply, relaxing wholesale banks' collateral constraint and allowing them to provide more attractive deposit claims. Consequently, more depositors switch from safe government bonds to deposit claims.

The key difference between a swap of reserves for ON-RRPs and a central bank balance sheet expansion in Section 3 is that the swap does not take out government bonds from the private sector. While the swap of central bank liabilities also leads to depositors switching to bank deposits, the magnitude of this switch is not as large as with a central bank balance sheet expansion, because, without taking out government bonds from the private sector, there is no direct pressure that makes bonds less attractive. Consequently, in contrast to the balance sheet expansion, this swap shifts the collateral condition upward as in Figure 4, implying an increase in transactions, or equivalently, a welfare improvement.

As before, the details of the welfare improvement can be understood through individual decisions. Firstly, a swap of reserves for ON-RRPs benefits depositors who trade with government bonds. While the supply does not change, this swap reduces the aggregate demand for government bonds because, by mitigating banking panics, fewer depositors flee to these safe assets. As a result, the interest rate on bonds increases ($\frac{\partial r^b}{\partial \bar{o}} > 0$), implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^b}{\partial \bar{o}} > 0$).

Secondly, a swap of reserves for ON-RRPs benefits depositors who trade with wholesale banks' deposit claims. Wholesale banks supply more deposit claims because, again, this swap relaxes their collateral constraint. Although this swap also increases the demand for these claims, the increased supply dominates the increased demand, implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^w}{\partial \bar{o}} > 0$). This result suggests that the increased demand in response to the swap is more moderate than central bank balance sheet expansions. The reason is that this swap puts no direct pressure to reduce the attractiveness of bonds, while balance sheet expansions lower their interest rate by reducing their supply. Furthermore, this moderate increase in demand for deposit claims puts little pressure on banks to compete for collateral to back them. However, as explained earlier, the effective collateral supply increases, implying higher real interest rates on collateral, such as reserves and interbank loans (i.e., $\frac{\partial r^\ell}{\partial \bar{o}} > 0$ and $\frac{\partial r^m}{\partial \bar{o}} > 0$).

Finally, a swap of reserves for ON-RRPs benefits retail bank depositors. The swap moves the composition of central bank liabilities away from reserves that only retail banks can hold, which could, in principle, reduce retail banking activities. Retail banks' asset holdings indeed have to fall, but so does their interbank borrowing. In fact, retail banks substitute their funding source from interbank borrowing to deposits because the former becomes more expensive in response to the swap (recall that $\frac{\partial r^\ell}{\partial \bar{o}} > 0$). As a result, the supply of retail banks' deposit claims increases, implying a higher trading volume for depositors who trade with them (i.e., $\frac{\partial c_2^r}{\partial \bar{o}} > 0$).

5 Conclusion

I study the implications of central bank balance sheet policies for wholesale banking panics, highlighting the crucial role of endogenous shifts in asset demand in response to central bank crisis interventions. A central bank balance sheet expansion and a swap of reserves for overnight reverse repurchase agreements mitigate wholesale banking panics. However, they have different, and sometimes unintended, implications on asset returns and, ultimately, welfare that is determined by transactions backed by assets. The expansion withdraws government bonds from the private sector, lowering bond returns and putting direct pressure on bonds, which makes them less attractive. In response, a large number of depositors shift to risky bank deposits, bidding down their returns. However, the swap leaves government bond supply unchanged and raises bond returns in equilibrium. Without the direct pressure that makes bonds less attractive, this swap generates only moderate shifts in asset demand, thereby raising (instead of lowering) deposit returns. In effect, I focus on the immediate consequences of these interventions through endogenous demand shifts, complementing previous studies that emphasize either the inefficient liquidation costs of banking panics or the long-run benefits of monetary easing.

References

Allen, F., Carletti, E., & Gale, D. (2014). Money, financial stability and efficiency. *Journal of Economic Theory*, *149*, 100–127.

Allen, F., & Gale, D. (1998). Optimal financial crises. *The journal of finance*, *53*(4), 1245–1284.

Andolfatto, D., Berentsen, A., & Martin, F. M. (2020). Money, banking, and financial markets. *The Review of Economic Studies*, *87*(5), 2049–2086.

Andolfatto, D., & Nosal, E. (2020). Shadow bank runs. *Working paper*.

Andolfatto, D., & Williamson, S. (2015). Scarcity of safe assets, inflation, and the policy trap. *Journal of Monetary Economics*, *73*, 70–92.

Arrata, W., Nguyen, B., Rahmouni-Rousseau, I., & Vari, M. (2020). The scarcity effect of qe on repo rates: Evidence from the euro area. *Journal of Financial Economics*, *137*(3), 837–856.

Bernanke, B. S. (2012). Some reflections on the crisis and the policy response. *Rethinking Finance: Perspectives on the Crisis conference*.

Bernanke, B. S. (2016). Should the fed keep its balance sheet large? *Ben Bernanke's Blog at Brookings*.

Bernanke, B. S. (2018). The real effects of disrupted credit: Evidence from the global financial crisis. *Brookings Papers on Economic Activity*, *2018*(2), 251–342.

Bush, R., Kirk, A., Martin, A., Weed, P., & Zobel, P. (2019). Stressed outflows and the supply of central bank reserves. Federal Reserve Bank of New York, Liberty Street Economics (20190220). https://ideas.repec.org/p/fip/fednls/87314.html

Campbell, J. Y. (1999). Asset prices, consumption, and the business cycle. In *Handbook of macroeconomics* (pp. 1231–1303, Vol. 1). Elsevier.

Cardamone, D., Sims, E., & Wu, J. C. (2023). Wall street qe vs. main street lending. *European Economic Review*, *156*, 104475.

Carlson, M., Duygan-Bump, B., Natalucci, F., Nelson, B., Ochoa, M., Stein, J., & Van den Heuvel, S. (2016). The demand for short-term, safe assets and financial stability:

Some evidence and implications for central bank policies. *International Journal of Central Banking*, *12*(4), 307–333.

Cooper, R., & Corbae, D. (2002). Financial collapse: A lesson from the great depression. *Journal of Economic Theory*, *107*(2), 159–190.

Cui, W., & Sterk, V. (2021). Quantitative easing with heterogeneous agents. *Journal of Monetary Economics*, *123*, 68–90.

Diamond, D. W., & Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, *91*(3), 401–419.

Diamond, D. W., & Rajan, R. G. (2006). Money in a theory of banking. *American Economic Review*, *96*(1), 30–53.

Fischer, S. (2016). The lender of last resort function in the united states. *Speech at the Committee on Capital Markets Regulation on February*, *10*.

Gertler, M., & Kiyotaki, N. (2015). Banking, liquidity, and bank runs in an infinite horizon economy. *American Economic Review*, *105*(7), 2011–2043.

Gertler, M., Kiyotaki, N., & Prestipino, A. (2016). Wholesale banking and bank runs in macroeconomic modeling of financial crises. In *Handbook of macroeconomics* (pp. 1345–1425, Vol. 2). Elsevier.

Gorton, G. (2008). *The panic of 2007* (tech. rep.). National Bureau of Economic Research.

Gorton, G. (2010). *Slapped by the invisible hand: The panic of 2007*. Oxford University Press.

Gorton, G., & Metrick, A. (2012). Securitized banking and the run on repo. *Journal of Financial economics*, *104*(3), 425–451.

Greenwood, R., Hanson, S., & Stein, J. (2016). The federal reserve's balance sheet as a financial-stability tool. *Innovative Federal Reserve Policies During the Great Financial Crisis*, 63–124.

Huang, X., & Keister, T. (2024). Preventing runs with redemption fees. *Working paper*.

Kim, K., Martin, A., & Nosal, E. (2020). Can the us interbank market be revived? *Journal of Money, Credit and Banking*, *52*(7), 1645–1689.

Lagos, R., & Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, *113*(3), 463–484.

Martin, A., McAndrews, J., Palida, A., & Skeie, D. R. (2019). Federal reserve tools for managing rates and reserves. *Working paper*.

Martin, A., Skeie, D., & Thadden, E.-L. v. (2014). Repo runs. *The Review of Financial Studies*, *27*(4), 957–989.

Ordoñez, G. (2018). Sustainable shadow banking. *American Economic Journal: Macroeconomics*, *10*(1), 33–56.

Robatto, R. (2017). Flight to liquidity and systemic bank runs. *Working paper*.

Robatto, R. (2019). Systemic banking panics, liquidity risk, and monetary policy. *Review of Economic Dynamics*, *34*, 20–42.

Robatto, R. (2024). Liquidity requirements and central bank interventions during banking crises. *Management Science*, *70*(2), 1175–1193.

Sengupta, R., & Xue, F. (2020). The global pandemic and run on shadow banks. *Main Street Views, Federal Reserve Bank of Kansas City, May*, *11*.

Williamson, S. D. (2019). Interest on reserves, interbank lending, and monetary policy. *Journal of Monetary Economics*, *101*, 14–30.

Williamson, S. D. (2022). Central bank digital currency and flight to safety. *Journal of Economic Dynamics and Control*, *142*, 104146.

Woodford, M. (2016). *Quantitative easing and financial stability* (tech. rep.). National Bureau of Economic Research.

Appendix

A Omitted Proofs

Proof of Lemma 1. Assume, on the contrary, that $\lambda^r = 0$. Then, from the equilibrium conditions (5) and (6) in the retail bank's problem, I obtain

$$r^\ell < r^b, \tag{A.1}$$

given that $u'(d^r) > 1$ because of the assumption regarding the scarcity of total government bond supply, which implies $d^r < c^\star$. However, from the equilibrium conditions (11) and (12) in the wholesale bank's problem,

$$r^\ell \geq r^b, \tag{A.2}$$

which is a contradiction. □

Proof of Lemma 2. Assume, on the contrary, that $b^w - [\rho + (1-\rho)\eta]\, b' > 0$. This implies that $\lambda^w = 0$. Then, from the equilibrium conditions (10) and (11),

$$u'\left(r^b b'\right) = 1 - \delta + \delta u'\left(d^w\right), \tag{A.3}$$

which implies

$$u\left(r^b b'\right) > u\left(d^w\right), \tag{A.4}$$

given that $u'(\cdot) > 0$ and $u''(\cdot) < 0$ and the fact that $u'(d^w) > 1$ because of the binding collateral constraint. Condition (A.4) further implies an equilibrium with full banking panic with $\eta = 1$. However, under $\eta = 1$, the collateral constraint (8) can never bind with $b^w - [\rho + (1-\rho)\eta]\, b' > 0$, which is a contradiction. □

Lemma A.1. *Conditions (14), (26) and (27) implicitly define a collateral condition $c_2^r = h\left(c_2^w\right)$ in the c_2^r-c_2^w space, where $h'\left(c_2^w\right) < 0$, i.e., the collateral condition is downward sloping in a partial banking panic equilibrium.*

Proof of Lemma A.1. Totally differentiating (14), (26) and (27) with respect to c_2^w, I obtain

$$u'\left(c_2^b\right)\frac{\partial c_2^b}{\partial c_2^w}-\delta u'\left(c_2^w\right)=0, \tag{A.5}$$

$$\alpha F_1'\left(c_2^r\right)\frac{\partial c_2^r}{\partial c_2^w}-\left(1-\alpha\right)\left(1-\rho\right)F_2\left(c_2^w\right)\frac{\partial \eta}{\partial c_2^w}+\left(1-\alpha\right)\left(1-\rho\right)\left(1-\eta\right)F_2'\left(c_2^w\right)=0, \tag{A.6}$$

$$\left(1-\rho\right)F_3\left(c_2^b\right)\frac{\partial \eta}{\partial c_2^w}+\left[\rho+\left(1-\rho\right)\eta\right]F_3'\left(c_2^b\right)\frac{\partial c_2^b}{\partial c_2^w}=0, \tag{A.7}$$

where $F_1\left(c\right)=c\left[1-\theta+\theta u'\left(c\right)\right]$, $F_2\left(c\right)=c\left[1-\theta\delta+\theta\delta u'\left(c\right)\right]$, and $F_3\left(c\right)=cu'\left(c\right)$. Note that

$$F_i'\left(c\right)>0, \quad \text{for} \quad i\in\{1,2,3\} \tag{A.8}$$

given that $-\frac{cu''(c)}{u'(c)}<1$ for all $c\geq 0$. Then, from (A.5)-(A.7), I have

$$\frac{\partial c_2^b}{\partial c_2^w}>0, \quad \frac{\partial \eta}{\partial c_2^w}<0, \quad \frac{\partial c_2^r}{\partial c_2^w}<0. \tag{A.9}$$

The last inequality $\frac{\partial c_2^r}{\partial c_2^w}<0$ implies that conditions (14), (26) and (27) implicitly define a function $c_2^r=h\left(c_2^w\right)$ with $h'\left(c_2^w\right)<0$. This is equivalent to say these conditions implicitly define a downward sloping collateral condition in the c_2^r-c_2^w space. □

Lemma A.2. *The no-arbitrage condition* (25) *implicitly defines a function:*

$$c_2^r=f\left(c_2^w\right), \tag{A.10}$$

where $f'\left(c_2^w\right)>0$, $\lim_{c_2^w\to 0}f\left(c_2^w\right)=0$, *and* $f\left(c^\star\right)=c^\star$.

Proof. The proof is trivial due to the diminishing marginal utility of consumption. □

Proof of Proposition 2. The goal is to show that $c_2^r=h\left(c_2^w\right)$ and $c_2^r=f\left(c_2^w\right)$ solve for a unique allocation $\left(c_2^w,c_2^r\right)$, where the first equation characterizes a downward-sloping collateral condition (Lemma A.1) and the second equation characterizes an upward-sloping no-arbitrage interbank market condition (Lemma A.2) in the c_2^r-c_2^w space. As illustrated in Figure 5, I can finish this proof by showing that $\lim_{c_2^w\to 0}h\left(c_2^w\right)-f\left(c_2^w\right)>0$ and $h\left(c^\star\right)-f\left(c^\star\right)<0$.

Confine attention to the collateral condition $c_2^r=h\left(c_2^w\right)$ first. Taking limit of $c_2^w\to 0$

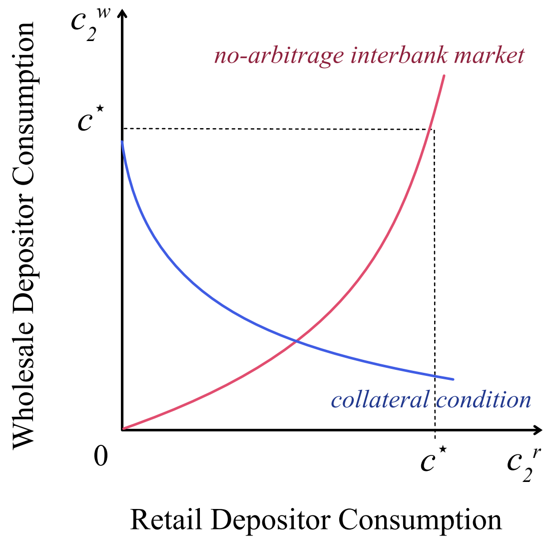

Figure 5: Existence and Uniqueness of Partial Banking Panic Equilibrium

on both side of (26), I have:

$$\theta\bar{m} = \lim_{c_2^w \to 0} \alpha c_2^r \left[1-\theta+\theta u'\left(c_2^r\right)\right], \tag{A.11}$$

as $\lim_{c\to 0} cu'(c) = 0$. For all $\bar{m} > 0$, this further implies, implicitly,

$$c_2^r = \lim_{c_2^w \to 0} h\left(c_2^w\right) > 0, \tag{A.12}$$

because $\lim_{c\to 0} cu'(c) = 0$ and $cu'(c)$ is increasing in c as $-\frac{cu''(c)}{u'(c)} < 1$. Furthermore, when $c_2^w = c^\star$, $c_2^r = h(c^\star)$ solves the following equation:

$$\theta\bar{m} = \alpha h\left(c^\star\right)\left[1-\theta+\theta u'\left(h\left(c^\star\right)\right)\right] + (1-\alpha)(1-\rho)(1-\eta)\,c^\star. \tag{A.13}$$

The expression $x\left[1-\theta+\theta u'(x)\right]$ is also strictly increasing in x. Therefore, for any feasible central bank's balance sheet size $\bar{m} \in \left(0, \hat{b}\right)$ with a fiscal policy $\hat{b}$ that satisfies Assumption 1, (A.13) implies

$$h\left(c^\star\right) < c^*. \tag{A.14}$$

To conclude, $\lim_{c_2^w\to 0} h\left(c_2^w\right) > \lim_{c_2^w\to 0} f\left(c_2^w\right) = 0$ and $h\left(c^\star\right) < c^* = f\left(c^\star\right)$. That is, $\lim_{c_2^w\to 0} h\left(c_2^w\right) - f\left(c_2^w\right) > 0$ and $h\left(c^\star\right) - f\left(c^\star\right) < 0$. A unique solution that $0 < c_2^w, c_2^r < c^\star$ exists by the Intermediate Value Theorem. Finally, $0 < c_2^b < c_2^w < c^\star$ because $u(c_2^b) =$

$\delta u(c_2^w)$ as in (14). $\square$

Proof of Proposition 3. As in Proposition 2, there is a unique partial banking panic equilibrium that can be solved by the no-arbitrage condition (25), collateral market clearing condition (26), bond market clearing condition (27), and condition (14). The proof for this proposition is performing comparative statics on the abovementioned system of equations with respect to an increase in reserve supply $\bar{m}$. For simplicity, consider a CRRA utility function

$$u(x) = \frac{x^{1-\sigma}}{1-\sigma}, \tag{A.15}$$

with $0 < \sigma < 1$, which satisfies assumptions on the utility function. Totally differentiating these functions with respect to $\bar{m}$, then solving the system of linear equations, I obtain

$$\frac{\partial c_2^w}{\partial \bar{m}} = -\frac{\Omega_3}{\delta \frac{u''(c_2^w)}{u''(c_2^r)}\Omega_1\Omega_5 + \Omega_2\Omega_5 + \Omega_3\Omega_4} < 0, \tag{A.16}$$

$$\frac{\partial \eta}{\partial \bar{m}} = \frac{1}{\Omega_3}\left(\delta \frac{u''(c_2^w)}{u''(c_2^r)}\Omega_1 + \Omega_2\right)\frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.17}$$

$$\frac{\partial c_2^r}{\partial \bar{m}} = \delta \frac{u''(c_2^w)}{u''(c_2^r)}\frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.18}$$

$$\frac{\partial c_2^b}{\partial \bar{m}} = \delta \frac{u'(c_2^w)}{u'\left(c_2^b\right)}\frac{\partial c_2^w}{\partial \bar{m}} < 0, \tag{A.19}$$

where

$$\Omega_1 = \alpha\left[1 - \theta + \theta(1-\sigma)u'(c_2^r)\right] > 0, \tag{A.20}$$

$$\Omega_2 = (1-\alpha)(1-\rho)(1-\eta)(1-\theta\delta) + (1-\alpha)(1-\sigma)\theta\delta u'(c_2^w) > 0, \tag{A.21}$$

$$\Omega_3 = (1-\alpha)(1-\rho)(1-\theta\delta)c_2^w > 0, \tag{A.22}$$

$$\Omega_4 = (1-\alpha)\left[\rho + (1-\rho)\eta\right](1-\sigma)\delta u'(c_2^w) > 0, \tag{A.23}$$

$$\Omega_5 = (1-\alpha)(1-\rho)\delta c_2^w u'(c_2^w) > 0. \tag{A.24}$$

Therefore, expanding the size of the central bank's balance sheet mitigates wholesale banking panics in a partial banking panic equilibrium. However, such a policy reduces consumption for all the depositors. $\square$

Proof of Proposition 1. The proof takes two steps. In the first step, I show there exist two critical values $\bar{m}_L$ and $\bar{m}_H$, which solve for $\eta = 1$ and $\eta = 0$, respectively, in conditions that determines partial panic equilibria (equations (25), (26), (27), and (14)). The withdrawal probability η is strictly decreasing in $\bar{m}$ in a partial banking panic equilibrium as in Proposition 3. Therefore, if such critical value exists, they satisfy $\bar{m}_L < \bar{m}_H$ and partial banking panic equilibria exits when $\bar{m} \in (\bar{m}_L, \bar{m}_H)$. I also show how these critical values change in response to a change in the probability of wholesale banking failure $1-\delta$. In the second step, I will show that a full banking panic exists when $\bar{m} < \bar{m}_L$ and a no banking panic equilibrium exits when $\bar{m} > \bar{m}_H$.

Step 1 First, consider a partial banking panic equilibrium with $\eta = 1$. Then, the critical value $\bar{m}_L$ and associated consumption allocation $\left(c_2^r, c_2^w, c_2^b\right)$ solve the following equations:

$$u'\left(c_2^r\right) = 1 - \delta + \delta u'\left(c_2^w\right), \tag{A.25}$$

$$\theta \bar{m}_L = \alpha c_2^r \left[1 - \theta + \theta u'\left(c_2^r\right)\right], \tag{A.26}$$

$$\hat{b} - \bar{m}_L = (1-\alpha) c_2^b u'\left(c_2^b\right), \tag{A.27}$$

$$u\left(c_2^b\right) - \delta u\left(c_2^w\right) = 0. \tag{A.28}$$

Taking the limit as $\bar{m}_L \to 0$, from (A.25) and (A.26), I have $c_2^r \to 0$ and $c_2^w \to 0$ because, in particular, $\lim_{c\to 0} cu'(c) = 0$. However, from (A.27), $c_2^b > 0$. Then, the last function (A.28) does not hold with equality. In fact, $u\left(c_2^b\right) - \delta u\left(c_2^w\right) > 0$ when $\bar{m}_L \to 0$. Similarly, taking the limit as $\bar{m}_L \to \hat{b}$, I obtain $c_2^b \to 0$ and $c_2^w > 0$. This implies that $u\left(c_2^b\right) - \delta u\left(c_2^w\right) < 0$ when $\bar{m}_L \to \hat{b}$. By Intermediate Value Theorem, these exists $0 < \bar{m}_L < \hat{b}$ which solves the above system of equations, and such $\bar{m}_L$ is unique by monotonicity.

Then, totally differentiating (A.25)-(A.28), with respect to δ, I can solve for

$$\frac{\partial \bar{m}_L}{\partial \delta} = \frac{1}{\theta} \alpha F_1'\left(c_2^r\right) \delta \frac{u''\left(c_2^w\right)}{u''\left(c_2^r\right)} \frac{\partial c_2^w}{\partial \delta} < 0, \tag{A.29}$$

where $F_1'(c) > 0$ with $F_1(c) = c\left[1 - \theta + u'(c)\right]$ (recall $-c\frac{u''(c)}{u'(c)} < 1$), and

$$\frac{\partial c_2^w}{\partial \delta} = -\frac{(1-\alpha)\theta F_2'\left(c_2^b\right) \frac{u\left(c_2^w\right)}{u'\left(c_2^b\right)}}{(1-\alpha)\theta F_2'\left(c_2^b\right) \delta \frac{u'\left(c_2^w\right)}{u'\left(c_2^b\right)} + \alpha F_1'\left(c_2^r\right) \delta \frac{u''\left(c_2^w\right)}{u''\left(c_2^r\right)}} < 0, \tag{A.30}$$

given $F_2'(c) > 0$ with $F_2(c) = cu'(c)$.

Similarly, consider a partial banking panic equilibrium with $\eta = 0$. Following the same procedure, I can show there exists a unique $\bar{m}_H$ that solves the following system of equations, and such $\bar{m}_H$ is strictly decreasing in δ.

$$u'\left(c_2^r\right) = 1 - \delta + \delta u'\left(c_2^w\right), \tag{A.31}$$

$$\theta \bar{m}_H = \alpha c_2^r\left[1 - \theta + \theta u'\left(c_2^r\right)\right] + (1-\alpha)(1-\rho)\, c_2^w\left[1 - \theta\delta + \theta\delta u'\left(c_2^w\right)\right], \tag{A.32}$$

$$\hat{b} - \bar{m}_H = (1-\alpha)\,\rho c_2^b u'\left(c_2^b\right), \tag{A.33}$$

$$u\left(c_2^b\right) - \delta u\left(c_2^w\right) = 0. \tag{A.34}$$

Step 2 First, when $\bar{m} = \bar{m}_L$, conditions (A.25)-(A.27) in a partial banking panic equilibrium are the same as the conditions solving for a full banking panic equilibrium. Consider a decrease in $\bar{m}$, c_2^w decreases and c_2^b increases, guaranteeing a full banking panic equilibrium as in condition (15). Similarly, when $\bar{m} = \bar{m}_H$, an increase in $\bar{m}$ results in a increase in c_2^w and a decrease in c_2^b, guaranteeing a no banking panic equilibrium as in condition (13). □

Proof of Proposition 4. Totally differentiating equations (14), (25), (32) and (33) with respect to $\bar{o}$, I obtain:

$$1 - \theta = \alpha F_1'\left(c_2^r\right)\frac{\partial c_2^r}{\partial \bar{o}} - (1-\alpha)(1-\rho)\, F_2\left(c_2^w\right)\frac{\partial \eta}{\partial \bar{o}} + (1-\alpha)(1-\rho)(1-\eta)\, F_2'\left(c_2^w\right)\frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.35}$$

$$0 = (1-\rho)\, F_3\left(c_2^b\right)\frac{\partial \eta}{\partial \bar{o}} + \left[\rho + (1-\rho)\,\eta\right] F_3'\left(c_2^b\right)\frac{\partial c_2^b}{\partial \bar{o}}, \tag{A.36}$$

$$u''\left(c_2^r\right)\frac{\partial c_2^r}{\partial \bar{o}} = \delta u''\left(c_2^w\right)\frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.37}$$

$$u'\left(c_2^b\right)\frac{\partial c_2^b}{\partial \bar{o}} = \delta u'\left(c_2^w\right)\frac{\partial c_2^w}{\partial \bar{o}}, \tag{A.38}$$

where $F_1(c) = c\left[1 - \theta + \theta u'(c)\right]$, $F_2(c) = c\left[1 - \theta\delta + \theta\delta u'(c)\right]$, and $F_3(c) = cu'(c)$. Note that $u'(c) > 0$, $u''(c) < 0$ and $F_i'(c) > 0$ for $i \in \{1, 2, 3\}$ given $-c\frac{u''(c)}{u'(c)} < 1$. From (A.37) and (A.38), $\frac{\partial c_2^r}{\partial \bar{o}}\frac{\partial c_2^w}{\partial \bar{o}} \geq 0$ and $\frac{\partial c_2^b}{\partial \bar{o}}\frac{\partial c_2^w}{\partial \bar{o}} \geq 0$, i.e., $\frac{\partial c_2^r}{\partial \bar{o}}$, $\frac{\partial c_2^w}{\partial \bar{o}}$, and $\frac{\partial c_2^b}{\partial \bar{o}}$ have the same sign of being positive or negative. From (A.36), $\frac{\partial \eta}{\partial \bar{o}}\frac{\partial c_2^b}{\partial \bar{o}} \leq 0$, i.e., $\frac{\partial \eta}{\partial \bar{o}}$ and $\frac{\partial c_2^b}{\partial \bar{o}}$ have different sign of being positive and negative. The only possibility of making condition (A.35) and (A.36) hold is

$$\frac{\partial \eta}{\partial \bar{o}} < 0, \frac{\partial c_2^r}{\partial \bar{o}} > 0, \frac{\partial c_2^w}{\partial \bar{o}} > 0, \frac{\partial c_2^b}{\partial \bar{o}} > 0. \tag{A.39}$$

□